7.2 Similar Polygons

Goal Identify similar polygons.

VOCABULARY

Similar polygons Two polygons are similar polygons if corresponding angles are congruent and corresponding side lengths are proportional.

Scale factor If two polygons are similar, then the ratios of the lengths of two corresponding sides is called the scale factor.

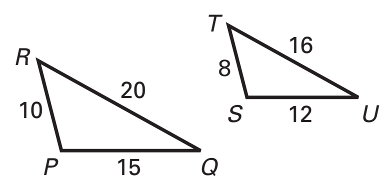
Example 1 *Use Similarity Statements*

$\triangle PRQ$ and $\triangle STU$ are similar.

a. List all pairs of congruent angles.

b. Write the ratios of the corresponding sides in a statement of proportionality.

c. Check that the ratios of corresponding sides are equal.



Solution

a. $\angle P \cong \angle S$, $\angle R \cong \angle T$, $\angle Q \cong \angle U$

b. $\dfrac{ST}{PR} = \dfrac{TU}{RQ} = \dfrac{US}{QP}$

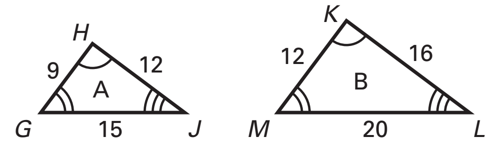
c. Write the ratios of corresponding sides and simplify.

$$\frac{ST}{PR} = \frac{8}{10} = \frac{4}{5}, \quad \frac{TU}{RQ} = \frac{16}{20} = \frac{4}{5}, \quad \frac{US}{QP} = \frac{12}{15} = \frac{4}{5}$$

The ratios of corresponding sides are all equal to $\dfrac{4}{5}$.

Example 2 *Determine Whether Polygons are Similar*

Determine whether the triangles are similar. If they are similar, write a similarity statement and find the scale factor of Figure B to Figure A.



Solution

1. Check whether corresponding angles are congruent.

From the diagram, $\angle G \cong \angle$ M, $\angle H \cong \angle$ K, and $\angle J \cong \angle$ L. So, the corresponding angles are congruent.

2. Check whether corresponding side lengths are proportional.

$$\frac{MK}{GH} = \frac{12}{9} = \frac{4}{3}, \quad \frac{KL}{HJ} = \frac{16}{12} = \frac{4}{3}, \quad \frac{LM}{JG} = \frac{20}{15} = \frac{4}{3}$$

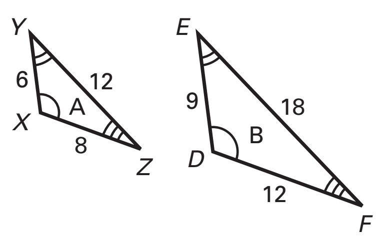
All three ratios are equal, so the corresponding side lengths are proportional.

Answer By definition, the triangles are similar. $\triangle GHJ \sim \triangle MKL$.

The scale factor of Figure B to Figure A is $\frac{4}{3}$.

✔ ***Checkpoint*** **Determine whether the polygons are similar. If they are similar, write a similarity statement and find the scale factor of Figure B to Figure A.**

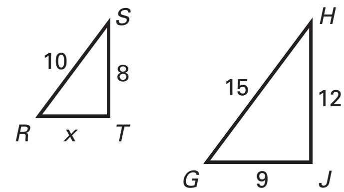
1.



Yes; $\triangle XYZ \sim \triangle DEF$; $\frac{3}{2}$

Example 3 *Use Similar Polygons*

$\triangle RST$ is similar to $\triangle GHJ$. Find the value of x.



Solution

Because the triangles are similar, corresponding side lengths are proportional.

$$\frac{GH}{RS} = \frac{JG}{TR}$$ **Write a proportion.**

$$\frac{15}{10} = \frac{9}{x}$$ **Substitute.**

$$(15)(x) = (10)(9)$$ **Cross product property**

$$15x = 90$$ **Multiply.**

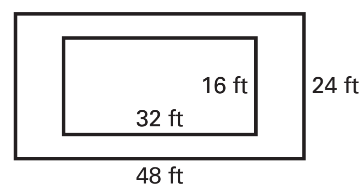
$$x = 6$$ **Divide each side by 15.**

Example 4 *Perimeters of Similar Polygons*

A pool and the patio around the pool are similar rectangles.

a. Find the ratio of the length of the patio to the length of the pool.

b. Find the ratio of the perimeter of the patio to the perimeter of the pool.



Solution

a. The ratio of the length of the patio to the length of the pool is

$$\frac{\text{length of patio}}{\text{length of pool}} = \frac{48 \text{ feet}}{32 \text{ feet}} = \frac{3}{2}.$$

b. Perimeter of patio $= 2(48) + 2(24) = 144$ feet

Perimeter of pool $= 2(32) + 2(16) = 96$ feet

The ratio of the perimeter of the patio to the perimeter of the pool is $\dfrac{\text{perimeter of patio}}{\text{perimeter of pool}} = \dfrac{144 \text{ feet}}{96 \text{ feet}} = \dfrac{3}{2}$.

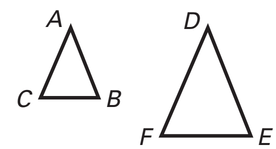
Follow-Up

In Example 4, what do you notice about the ratio of the lengths and the ratio of the perimeters?

The ratios are equal.

THEOREM 7.1: PERIMETERS OF SIMILAR POLYGONS

Words If two polygons are similar, then the ratio of their perimeters is equal to the ratio of their corresponding side lengths.

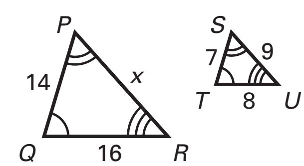


Symbols If $\triangle ABC \sim \triangle DEF$, then

$$\frac{DE + EF + FD}{AB + BC + CA} = \frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}.$$

✔ ***Checkpoint*** In the diagram, $\triangle PQR \sim \triangle STU$.

2. Find the value of x.



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3. Find the ratio of the perimeter of $\triangle STU$ to the perimeter of $\triangle PQR$.

$\frac{2}{1}$